



Examiners' Report Principal Examiner Feedback

January 2023

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2 (WFM02)
Paper 01

General

This paper proved to be a good test of candidates' ability on the WFM02 content and plenty of opportunity was provided for them to demonstrate what they had learnt. Marks were available to candidates of all abilities. The questions that proved to be the most challenging were 5, 6, 8 and 9.

Presentation was generally good and candidates often showed sufficient working to make their methods clear. There were some exceptions such as in question 7(b) where values sometimes appeared with no justification with a distinct lack of method shown. This was presumably due to candidates resorting to their calculators without explaining what they were doing. This sometimes occurred in question 1(b) where candidates, having found their derivatives, jumped straight to a Maclaurin expansion with no evidence where the values had come from. This can be a risky strategy if the values are incorrect as examiners are unable to judge if the method is correct.

Report on Individual Questions

Question 1

Although many candidates were able to score full marks on this question, there were a significant number of students who lost marks for a variety of reasons. It was surprising that many candidates were inaccurate with the differentiation in part (a).

In part (a) candidates were able to at least obtain the method mark for the correct form of the first derivative. However, many candidates differentiated $\ln(5 + 3x)$ to $\frac{1}{5+3x}$ rather than $\frac{3}{5+3x}$. Some then applied the quotient rule to find the subsequent second and third derivatives rather than using the chain rule. As a result, sign and arithmetic errors were then seen.

In part (b), most knew they had to find the values of the derivatives at $x = 0$ and proceeded to apply the Maclaurin series correctly. A significant number of candidates did not use "Hence" and wrote $\ln(5 + 3x)$ as $\ln 5(1 + \frac{3}{5}x)$ and used the standard result in the formula book to establish the expansion.

The B mark in part (c) was scored by many, particularly as a follow through was allowed from part (b) provided the signs were changed on the appropriate terms. A significant number of candidates started again with $\ln(5 - 3x)$ and found the first 3 derivatives and so replicated the work already done in parts (a) and (b). Credit was allowed in such cases, provided the final expansion was correct.

In part (d), candidates who realised that all that was required was the subtraction of their 2 series, could obtain the method mark. Earlier errors often meant that the accuracy mark became unavailable to many candidates.

Question 2

This was an expected and routine question and was accessible to the vast majority of candidates.

Part (a) was well answered by most, where the median score was 2. Very few students scored 0 marks here, and of those who scored 1 mark, carelessness with basic algebra or arithmetic was usually the cause. Some

candidates used a correct partial fraction method but just stated the values of their constants rather than stating the fractions themselves.

Part (b) was not as successful, although many candidates still scored full marks. The method of differences was well understood by most candidates, and many went on to achieve full marks. Some candidates did arrive at the final answer, but did not show sufficient working, so did not score all available marks. A small number of candidates forgot to multiply their fraction by $1/8$ while some candidates extracted the incorrect terms from the differences. A common issue that led to errors was when candidates multiplied the factor of $1/8$ into the fractions before cancelling terms which made it much more difficult to achieve the final solution. In general, if candidates obtained the correct partial fractions, then most were able to obtain the correct non-cancelling terms. When combining fractions in part (b), problems sometimes arose from not finding a lowest common denominator leading to unwieldy algebra and incorrect answers.

Question 3

In part (a), the majority of candidates could apply the chain rule correctly to obtain $\frac{dy}{dx} = -\frac{1}{z^2} \frac{dz}{dx}$ and proceeded to substitute into the differential equation and obtain the printed result with no errors. Some opted for more convoluted approaches such as $y = \frac{1}{z} \Rightarrow zy = 1 \Rightarrow y \frac{dz}{dx} + z \frac{dy}{dx} = 0$ and then made the substitution. A small minority used the substitution to find $\frac{dz}{dx}$ and then substituted into the second differential equation to show that it gave the first differential equation.

In part (b), most knew that that an integrating factor was required and attempted $e^{-\int \frac{1}{x} dx}$. A surprising number of candidates interpreted this as $-x$ rather than $\frac{1}{x}$. Regardless of this, the majority knew how to apply the integrating factor and could obtain the method mark. Of those with the correct equation after integration of $\frac{z}{x} = \frac{1}{x^2} + c$, a surprising number then wrote $z = \frac{1}{x} + c$.

Almost all knew to reverse the substitution in part (c) to find the constant of integration although there were a significant number of errors when inverting fractions and dealing with the resulting algebra. It was disappointing at this level to see candidates writing the reciprocal of $\frac{1}{x} + cx$ as $x + \frac{1}{cx}$.

Question 4

This question was a good source of marks for the majority of candidates and many fully correct solutions were seen.

In part (a), the vast majority of the candidates correctly found the second derivative and then recognised the need to use the Product Rule to determine the third derivative. The main difficulties encountered were in finding the fourth derivative and the associated handling of the $2\left(\frac{dy}{dx}\right)^2$ term.

A number of the cohort thus incorrectly obtained $B = 4$. A small minority of candidates obtained a second derivative of $2y \frac{dy}{dx}$ rather than $2y \frac{dy}{dx} - 1$ but were allowed to recover in this part of the question.

In part (b), nearly all of the candidates used a correct method to obtain the Taylor series solution for y . Some of the candidates incorrectly obtained a series in ascending powers of x but in general, the main errors came from incorrect coefficients, resulting from errors made in part (a) when differentiating.

Question 5

This question was well attempted by the vast majority of candidates although full marks were rare. Most were able to find at least two critical values (3 and $-19/3$), by multiplying both sides by $(x + 8)$ with many candidates gaining at least two marks for this approach. However, there were also a large number of candidates who did not know how to correctly deal with the modulus term. It was not uncommon to see one inequality/equation and no consideration of the case where the modulus could be replaced with $-(x + 8)$. It was also surprising that there were many candidates who did not consider -8 as a critical value, and therefore scored the initial B0 and final A0. If candidates successfully found all 3 critical values, most were able to score at least one of the final two accuracy marks for two correct regions, but it was much less common to see a candidate score full marks. Methods involving common denominators and squaring both sides were less common, as were solutions given in set notation. In some rare cases, candidates attempted to cancel factors by recognising that $6 - 2x = 2(3 - x)$ but such approaches usually resulted in errors with the resulting algebra.

Question 6

This was a demanding question of its type, particularly in part (b).

In part (a), it was pleasing to see that the majority of the candidates recognised the locus of P as being a perpendicular bisector although a number of the sketches lacked clarity and conciseness. It would, in particular, be useful for candidates to indicate, in their sketch, which part is the required locus.

In part (b), nearly all of the cohort correctly obtained z in terms of w in order to make progress in this part of the question. For those that went on to use Way 1 in the mark scheme, many introduced $w = u + iv$ or $w = x + iy$ and went on to process z into its real and imaginary parts by substituting into a Cartesian equation of the locus of P . However, the algebra that followed thereafter defeated many and thus the final form required was rarely seen. There was also some misconception of what to substitute, with some candidates only substituting the numerators of the real and imaginary parts. Some candidates substituted directly into the locus of P but such attempts were often aborted because of the resulting algebraic manipulation that was required. For those that went on to use Way 2 in the mark scheme, the vast majority correctly substituted into the given locus of P and proceeded to obtain the first 3 marks. However, the subsequent substitution of $w = u + iv$ and use of Pythagoras caused numerous difficulties and as in the case above, the final form required was rarely seen.

Way 3 in the mark scheme was the least seen method used although those who adopted this approach could at least score the first 3 marks. Having reached $|w(2i - 3) + 3i| = 2$, students were then unsure how to proceed and some treated this as an equation without the modulus signs in an attempt to get it in the form required in the question.

Question 7

In part (a), most candidates realised they needed the real part of the expansion of $(\cos x + i \sin x)^5$ and extracted the required terms, although a small number omitted the binomial coefficients. A small minority of candidates did not use de Moivre and opted instead to use multiple angle formulae. A smaller minority chose the more circuitous route of using the $\left(z + \frac{1}{z}\right)^n$ form with varying degrees of success. Many could obtain the correct expression in the form required although there were some slips with signs and coefficients.

Part (b) was left unanswered by some candidates. Those who attempted this part often saw the link with part (a) and could simplify the equation to obtain a quadratic equation in $\sin \theta$. Many then solved this equation, often using a calculator, but sometimes forgot to square root the answers to obtain values for $\sin \theta$ rather than $\sin^2 \theta$. Examiners commented on the lack of working being shown in this part. A correct quadratic equation in $\sin \theta$ would sometimes be followed by incorrect values for either $\sin^2 \theta$ or $\sin \theta$ and in some cases, candidates went straight to values of θ with no indication that a correct method was being adopted. Candidates are advised to show all their working to avoid losing marks unnecessarily.

Question 8

In part (a), the vast majority of the candidates recognised the need to find the stationary point of the function $y = r \sin \theta$ and correctly applied the product rule to find the polar coordinates of point P . There were some unnecessary attempts to use a double angle identity on $\sin^2 \theta$ in order to differentiate although these were usually successful. A few candidates stopped having found the correct value of θ and did not give a value for r as requested.

In part (b), many of the cohort identified the need to find an area bounded by the polar curve and combine this with the area of an associated triangle. The integration required for the polar curve was often completed correctly with a sign error on the double angle formula for $\cos 2\theta$ being the main reason for any loss of marks. Moreover, the vast majority of the candidates used the limits of integration as $\pi/6$ and 0 and achieved much success in evaluating the integral. Those students who used limits of $\pi/2$ and 0 encountered more difficulties and struggled to recognise how the area given by this integral helps to find the area of R . The associated triangle was, in most cases, dealt with confidently although there were a few cases where there was a distinct lack of working for this particular area. The final answer was correctly obtained by many of the cohort although there were a number of candidates who lost marks at the end of their solution when manipulating the terms involving π and $\sqrt{3}$.

Question 9

This was not a particularly successful question with very few candidates securing full marks. The candidates were split into those who knew and largely could apply the standard methods required for most of the question and some who were under prepared for work at this level and consequently scored partial marks.

Part (a) (i) was quite well done with many reaching the required form with a minority obtaining the correct solution with the first derivative in terms of x , and not y and t as requested. Some who started correctly had difficulties with finding the reciprocal of $\frac{1}{2}t^{-\frac{1}{2}}$ when applying the chain rule. Part (a) (ii) was not as successful, with many incorrect/partially correct responses seen. Some differentiated their answer to (a)(i) with respect to

t rather than x meaning they lost all the marks for having no expression of the required form. It was common amongst the incorrect responses to see a factor of $\frac{dt}{dx}$ omitted – again as a result of not knowing how to apply the chain rule correctly. Candidates who had worked with x and t in part (a)(i) found a simpler expression to differentiate in (a)(ii) using the product and chain rules, but few who used this approach were able to convert back into y and t at the end.

In part (b) most candidates could score at least the method mark for substituting their expressions into the differential equation and replacing the x terms. Some candidates ‘jumped’ from the substitution to the final answer with no intermediate line of working and lost the accuracy mark. Candidates should be reminded that full working is required in ‘show that’ questions, including any intermediate steps taken to achieve a printed answer.

Part (c) was the most successful part of the question, with many fully correct solutions seen. Most were able to solve the auxiliary equation and proceed to the correct complementary function, but some candidates lost marks through poor algebra or using the wrong form of the complementary function. It was common to see a hybrid of t and x terms throughout this question, but a complementary function in terms of x was condoned here. The most common error after finding the complementary function was to use the incorrect form for the particular integral, with many students attempting a particular integral of the form $y = at$ rather than $y = at + b$. This led to an incorrect particular integral and a loss of three marks. Other candidates used a quadratic form for the particular integral, and although unnecessary, was a valid form which could be differentiated and used to obtain the general solution in terms of t . Common errors were in failing to write the final solution as ‘ $y =$ ’ or failing to write the general solution in terms of t not x .

Most were able to get the B1ft mark at the end in part (d) for replacing their t with x^2 if a general solution had been found in part (c).